BUS INCREMENTAL COSTS AND ECONOMIC DISPATCH

J.S. Luo
Northeast Electric Power Institute
Jilin, PROC

E.F. Hill
Member
University of New Brunswick
Fredericton

T. H. Lee
Senior Member
The NB Electric Power commission
New Brunswick, CANADA

ABSTRACT

Various analytical techniques have been developed for the economic dispatching of generation to meet system load. A recently proposed approach to economic dispatching, which has bus incremental costs as the key variables, offers a possibility to get an insight into the economic dispatch mechanism. This paper attempts to derive and promote some novel concepts showing how bus incremental costs can be viewed as a "potential" in a dispatch analog of the power system, such that basic electrical network solution techniques can be readily applied to solve the economic dispatch problem.

INTRODUCTION

Various analytic techniques have been developed for the calculation of power system economic dispatching to allocate system load on each generator operating in the system. However, the existing approaches, relying on coordination [1] or on optimal load flow [2, 3], have some difficulties to expose the detailed mechanism of economic dispatching. The present coordination approach has difficulty in detailed transmission loading determination, hence the precise transmission loss evaluation. For the optimum loadflow, the effects of system parameter variations on economic operation may be obscured by the complexity of modelling or the complexity of solution techniques.

The recently proposed network approach [4], which has bus incremental costs as the key variables, offers a possibility to get an insight into the economic dispatching mechanism. The authors have found that the conditions of economic operation can be simply and explicitly expressed through the use of the bus incremental cost. The network model of dispatch, through the concept of the bus incremental cost, not only gives quantitative results, but also derives some useful new concepts in economic operation. It depicts a clear picture of economic operation of the whole system and enables one to estimate the influence of various parameter variations, and to apply the electrical network solution techniques easily to handle the economic dispatch problem.

To clearly demonstrate these ideas the paper is structured in the following way. Bus incremental costs are first defined and the relationships are established between bus incremental costs, and the following: generator incremental costs, line transmitted power, and penalty factors. The loop potential equations are added, and transmission line limits are included. Finally, it is shown how bus incremental costs may be used in system planning.

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Following from the pattern which evolved in the decoupled load flow, the intention of this research is to first deal with active power allocations and later to deal with the reactive power allocation. Ultimately it is visualized that these decoupled processes may be separately placed on a dual microprocessor system for solutions and interaction as necessary.

BASIC NETWORK MODEL FOR ECONOMIC DISPATCH

The system model around which the concepts of this paper are developed is one in which the variables are analogs of the variables of basic network theory. Since the detailed demonstration of the analogy is given in [4], only a summary is given here.

In all practical cases, the production cost of generator i can be represented as a quadratic function of G_i,

$$F_i(G_i) = a_i G_i^2 + b_i G_i + c_i \tag{1}$$

The objective function F to be minimized is the total system production cost

$$F = \sum^{NG} F_i(G_i) \tag{2}$$

where NG is the number of dispatchable generating units in the system.

Elgerd [7] presented an equation which, under the assumption of 1.0 per unit voltage at each end of the line, suggested that losses on a transmission line might be represented as the product of line resistance and the square of the transmitted power on the line. In a model which has been validated many times [4, 6], half of the line losses may be lumped at each of the two terminating buses as shown in Figure 1.

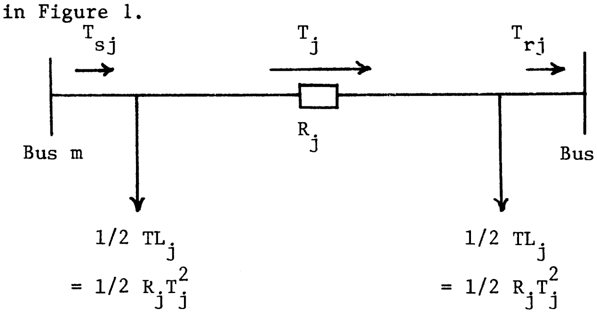


Fig. 1 Line Loss Model

The transmission power T_{sj} of line j at its sending end, therefore, has the form of a transmitted component and a loss component

$$T_{sj} = T_j + 1/2\ TL_j = T_j + 1/2\ R_j T_j^2 \tag{3}$$

where T_j is the average transmitted power component of line j; TL_j is the loss component of line j; and R_j is the resistance of line j.

Each bus in a power system may have one or more generators, an active power load D, along with any number of transmission lines connected to it. The power balance equation at each bus k therefore becomes the equality constraint ϕ_k at that bus.

$$\phi_k = \sum_{i(k)} G_i - D_k - \sum_{j(k)} (T_j + 1/2\ R_j T_j^2) = 0 \tag{4}$$

where D_k is the active load at bus k, and i(k) and j(k) denote that the summations are respectively over the generators i and the lines j which are connected to bus k.

The Lagrangian, based on standard calculus, is

$$L = F - \sum^{NB} \alpha_k \phi_k \tag{5}$$

where NG is the number of dispatchable generation units in the system; NB is the number of buses; $F_i(G_i)$ is the production cost of generator i expressed in terms of its active power loading G_i; ϕ_k is the active power balance equation at bus k as the equality constraint; and α_k is the Lagrangian multiplier associated with the equality constraint.

The model presented above, in Eqns. (2), (4), (5), consists of an objective function plus equations of equality constraints. Reference [4] shows that, when the optimality conditions are imposed, a series of equations result which have bus incremental costs as variables. Furthermore, these equations have a form which allows them to be modelled in a conventional network form consisting of impedances, voltage sources, and current sources. The various network sections will be shown in the development of the paper.

BUS INCREMENTAL COSTS (BIC)

A system incremental cost has long been a significant factor in economic dispatching of power systems. This is based on a concept of biasing the generator incremental costs (GIC's) by so called 'penalty factors' such that a unique value of system incremental cost, α, at each generator terminal, which is connected to the equivalent load bus, can be achieved. Beyond this single value, however, the availability of bus incremental costs (BIC's) at all buses of the power system will provide a rich insight into the actual dispatching process taking place within the power system. As well, they provide numerical values for several performance indices which are necessary for proper economic decisions in the operation of the system.

The Lagrangian multiplier α_k, as documented by Peschon et al. [5] has the physical meaning of the change in system production cost with respect to unit load variation at bus k, i.e.

$$\alpha_k = \frac{dF}{dD_k} \tag{6}$$

Hereafter it will be termed the bus incremental cost (BIC). In the network model for economic dispatch, BIC is the counterpart of node potential, i.e. economic potential, which decides the economic generation schedules and powerflow in the network. By means of the proposed network approach, all the BIC's of the power system including those at passive or load buses can be solved for directly.

The material which follows is an elaboration on the significance of bus incremental costs (BIC) in the understanding, solution, and application of the economic dispatching process, when some inequality constraints are considered.

BIC-GIC RELATIONSHIPS

It is important to distinguish the bus incremental cost (BIC), α_k, from the generator incremental cost (GIC), dF_i/dG_i or λ_i. BIC exists at each bus and measures the system incremental cost with respect to the load variation at the bus, whereas GIC exists at each generator and measures the incremental production cost of the generator with respect to its loading level. They are two different concepts, although closely related to each other.

One of the optimality conditions is derived from the Kuhn-Tucker theorem by differentiation with respect to generator power G of the function L augmented to include the effect of generator capacity limits. It establishes the connection between BIC and GIC as

$$\frac{dF_i}{dG_i} + W_i = \alpha_k \tag{7}$$

where W_i is the Kuhn-Tucker multiplier for generation limitation at generator i, and

$$W_i > 0,\ \text{when } G_i = G_{imax} \tag{8a}$$

$$W_i = 0,\ \text{when } G_{imin} < G_i < G_{imax} \tag{8b}$$

$$W_i < 0,\ \text{when } G_i = G_{imin} \tag{8c}$$

where G_{imax} and G_{imin} are, respectively, the upper and the lower limits of generation G_i, respectively. In Equations (7) and (8), i denotes any generator connected to bus k.

Upon substitution of λ_i as the generator incremental cost this becomes

$$\lambda_i + W_i = \alpha_k \tag{9}$$

Eqn (9) shows that the BIC, α_k, may not necessarily equal the GIC, λ_i, even when generator i is connected to bus k.

For completeness, upon substitution of the production cost form, one gets

$$2a_i G_i + b_i + W_i = \alpha_k \tag{10}$$

The difference between BIC and GIC, which is the value of the Kuhn-Tucker multiplier W_i, indicates the saving of system production cost which can be gained if the effective limitation, upper or lower, is slackened by one megawatt. Eqn. (7) shows that the economic loading condition for any generator is totally decided by its terminal BIC. The functional dependence of G_i upon α_k can be expressed as follows:

$$G_i = G_{imax} \quad \text{when} \quad \alpha_k \geq \lambda_{imax} \tag{11a}$$

$$G_i = (\alpha_k - b_i)/2a_i \quad \text{when} \quad \lambda_{imin} < \alpha_k < \lambda_{imax} \tag{11b}$$

$$G_i = G_{imin} \quad \text{when} \quad \alpha_k \leq \lambda_{imin} \tag{11c}$$

where

$$\lambda_{imax} = 2a_i G_{imax} + b_i;$$

$$\lambda_{imin} = 2a_i G_{imin} + b_i.$$

Referring to Equation (11b), it is seen that there is a flow quantity G_i which is equal to the difference of two quantities (which may be taken as the analog of potentials) divided by a constant (which may be taken as the analog of resistance). The network model associated with equation (11b) is shown in Figure 2(b). To cause this to fit the standard circuit network form, the two potential quantities have to be negated.

Referring to equations (11a) and (11c), it is seen that there is a flow quantity G_i which is at the fixed values of G_{imax} and G_{imin} respectively. This is the analog of a constant current source, as indicated by the open arrows in the circle in Figs 2(a) and 2(c).

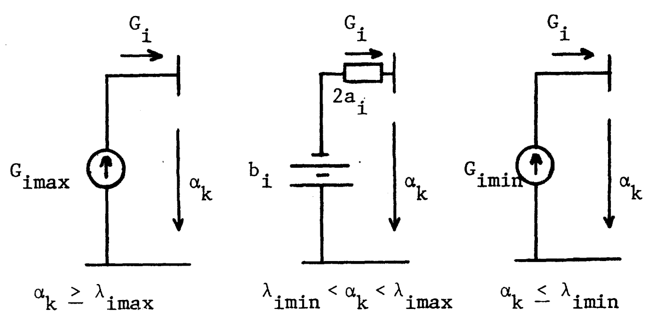


Fig. 2 Network models of generator (refer to Eqn. 11)

Eqn (11b) and Figure 2(b) make an interesting point. The b_i term of the generating cost characteristic appears as an internal EMF in much the same way as the internal voltage of a battery. The term 2a appears as a resistance. A second interesting feature in the network model for generation sources can also be seen from Eqn (11b). The generator power flow is from the lower 'EMF', b_i, to a higher terminal potential, α_k. This is contrary to the common circuit theory. Also, the constrainted generation, either due to a high GIC (Eqn 11(c)) or low capacity (Eqn 11(a)) situation, merely changes from a potential source (or Thevenin equivalent) to a current source (or Norton equivalent).

Eqns.(8) and (11) show, therefore, the interaction between BIC and GIC. When the BIC's of a power system (α_k) have been solved for, all the generation schedules can be calculated immediately.

BIC-TRANSMITTED POWER RELATIONSHIP

A second optimality condition establishes the relationship between BIC and transmitted power (Figure 2). This is obtained by differentiating the function L, given in Equation (5), with respect to T_j. In considering L, it is first noted that F is not a function of T_j. Further in the α_k, ϕ_k component of L, T_j appears only in the two terms associated with each of the two terminating buses for line j. They are, as shown in Fig. 1,

$$\alpha_m(T_j + 1/2 R_j T_j^2)$$

for bus m, and,

$$\alpha_n(-T_j + 1/2\, R_j T_j^2)$$

for bus n. Therefore,

$$\frac{\partial L}{\partial T_j} = \alpha_m + \alpha_m R_j T_j - \alpha_n + \alpha_n R_j T_j = 0 \text{ or}$$

$$\alpha_n - \alpha_m = R_j T_j\,(\alpha_n + \alpha_m) \tag{12}$$

Eqn (12) can be interpreted from two points of view.

1. Huskilson expressed the same relation in [6] as

$$\alpha_n - \alpha_m = 1/2\,(\alpha_n + \alpha_m)\,\frac{d(R_j T_j^2)}{dT_j} \tag{13}$$

and gave the interpretation that the incremental transmission loss, $d(R_j T_j^2)/dT_j$, should be charged at the average of its two terminating bus BIC's, $1/2(\alpha_n + \alpha_m)$. The difference of two adjacent bus BIC's is incurred by the cost of incremental transmission loss in the line between them.

2. Define the transmission incremental cost coefficient, TICC, of line j as:

$$RH_j = R_j(\alpha_n + \alpha_m) \tag{14}$$

and rewrite Eqn (14) as

$$\alpha_n - \alpha_m = T_j\, RH_j \tag{15}$$

With BIC as the counterpart of node potential and power flow as that of current, TICC will naturally be the counterpart of the Ohm's law in economic dispatch as shown in Figure 3. However, TICC is a nonlinear 'resistance' because its value, RH_j, depends upon the values α_n and α_m. The direction of power flow in this network model of the transmission line, contrary to the electric circuit theory, is from the lower 'potential' bus to the higher 'potential' bus.

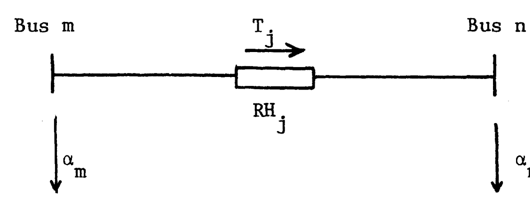


Fig. 3 Transmission power T and its terminating buses m and n

Eqn (15) shows that in a transmission network, along the path of positive transmitted power the BIC's encountered get higher and higher, and the difference of the two BIC's of adjacent buses is equal to the power flow on the line between them multiplied by the TICC of the line.

Conversely, rewrite Eqn (12) as

$$T_j = \frac{\alpha_n - \alpha_m}{R_j(\alpha_n + \alpha_m)} \tag{16}$$

It shows that, if the BIC's of two adjacent buses are different, the difference will drive an economic power flow from the lower BIC bus to the higher BIC bus. The value of the flow is proportional to the difference in BIC's of the two adjacent buses, and inversely proportional to the line resistance and the average of these two BIC's.

As an aside at this point, it should be noted that the Eqn (15) form of Eqn (12) works very well in an iterative solution process since the α_n and α_m values from a previous iteration can be used to calculate a value of RH_j to be used in the current iteration of the solution process.

BIC AND PENALTY FACTORS

The penalty factor method has been widely used for dispatching by power utilities for three decades. The relationship between penalty factor and BIC is now investigated. By the definition of penalty factor [1]

$$PF_i = \alpha_s / \frac{dF_i}{dG_i} \tag{17}$$

where PF_i represents the penalty factor of generator i connected at bus k, and α_s is the BIC of the reference bus.

It has been shown that the GIC of a dispatchable generator is equal to the BIC of its terminal bus if the generator capacity limitations are not violated. That is,

$$\lambda_i = \alpha_k \tag{18}$$

As a result, the penalty factor

$$PF_i = \alpha_s / \alpha_k \tag{19}$$

So, the penalty factor of a generator can be expressed as the ratio of the BIC of the reference bus to that of the generator bus. Consequently, after all the BIC's of a system are solved via the network approach, the penalty factors of all generators in the system, no matter which bus is appointed to be reference bus, can be calculated immediately.

TRANSMISSION POWER WITH LOOP CONSTRAINT

A casual inspection of Eqns (1)-(5) reveals a very interesting phenomenon, which is that there is no potential equation in the model equivalent to a general Kirchhoff's potential law. The question immediately comes up as to how any accurate line flows were obtained from this model.

This leads to an interesting observation. The dispatch model shows that line flows, based on economic cost considerations, are determined by the resistance and BIC's of the line. The load flow model shows that line flows are determined largely by the reactance and voltage angles of the line. Consequently, in theory, results obtained from the dispatch model and results obtained from the load flow model would only compare well if there were fairly consistent X/R ratios for the lines and transformers in the system. In practice, there seems to be a compensating effect within the loops of a power system so that generally comparable results are obtained even if the X/R ratios are dissimilar. However to be completely general, non-uniform X/R ratios must be incorporated into the dispatch model.

When there are non-uniform X/R lines in a loop, a circulating flow will exist in the loop to balance the loop voltage difference according to the Kirchhoff's potential law. The calculation of line flows of optimal operation has to take this situation into consideration.

The problem can be solved by including the loop constraint in the Lagrangian. The constraint is expressed as the fact that the sum of voltage phase angle differences around a closed loop is equal to zero. An approximation of the voltage phase angle across a line is the product of line reactance and line power. The loop constraint is

$$\psi = \Sigma X_j T_j = 0 \tag{20}$$

where the summation is over all the branches which constitute the loop.

The Lagrangian becomes

$$L = \sum^{NG} F_i(G_i) - \sum^{NB} \alpha_k \phi_k - \mu \psi \tag{21}$$

The condition for economic generation schedules is the same as specified by Eqn (7) or Eqn (11). The condition for economic power flow on line j included in the loop becomes

$$\alpha_n - \alpha_m = T_j R_j (\alpha_n + \alpha_m) - \mu X_j \tag{22}$$

where μ is the Lagrangian multiplier for the loop constraint. It should be noted that this formulation is easily made general, but only one loop is considered for purposes of illustration as shown in Fig. 4.

The multiplier μ has a physical meaning which is that it is the sensitivity of the system production cost with respect to the angle constraint in the closed loop. It shows how much can be saved if a phase-shifter is installed in the loop with the phase shift angle equal to one radian. It also can serve as an index to indicate the effect caused by the non-uniformity of X/R ratios of lines in the loop.

Rewrite Eqn (22) in the form of Eqn (16)

$$T_j = \frac{\alpha_n - \alpha_m}{RH_j} + \frac{\mu X_j}{RH_j} \tag{23}$$

Comparing it with Eqn (16), one can see that the multiplier μ regulates power flows in the loop so as to arrive at a loop constrained optimum.

The network model of power flows in a closed loop is shown in Figure 4. The μX_j term in each branch can be interpreted as a dependent EMF controlled by μ.

INEQUALITY CONSTRAINT OF TRANSMISSION POWER

The network formulation of the economic dispatch problem allows for an interesting treatment of transmission line flow limits. When a line flow violates its limit $\overline{T}$, it can be represented by a constant current branch with value $\overline{T}$ in the network model. If the branch is included in a closed loop, the dual variable μ will provide the necessary balancing effect to regulate the power flows in the other lines of the loop such that the loop constraint of Eqn (20) is observed.

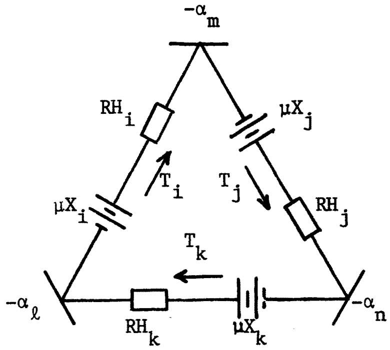


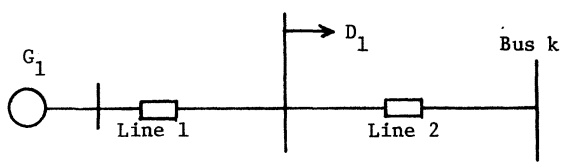
Fig. 4 Network model of power flows in a closed loop

BIC AND SYSTEM PLANNING

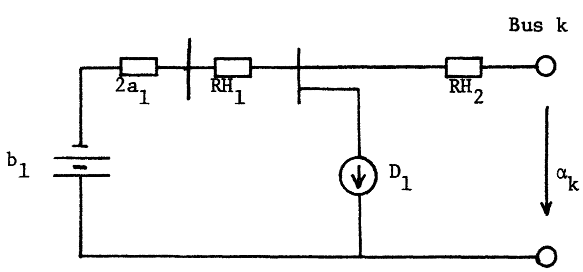
BIC, besides its application in economic dispatch, can also be used to assess the economic effect of system planning.

It has been shown in previous sections how components of the economic dispatch can be represented by network analogs. Because this is the case, it follows that any network methodology or theorems should be applicable. Thevenins Theorem is, of course, one of these.

As an illustrative example, consider Figure 5(a) showing a small power system which consists of a generator, two lines, three buses, and a load D . It can be represented as an economic dispatch network as shown in Figure 5(b) based on the concepts given in Figures 2 and 3, and recognition of the fact that a load appears as a current source in the economic dispatch network analog.



(a) Small Power System



(b) Economic Dispath Network Analog
Fig. 5 An Example System for Network Analogy

In the network analog, the nonlinearities of the RH factors are neglected, and system losses are not shown.

By Thevenins Theorem, looking in at bus k the economic dispatch network can be replaced by a source α_{ko} and a series resistance H as shown in Figure 6.

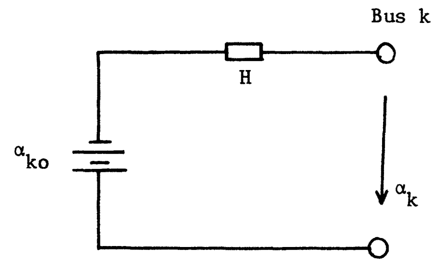


Fig. 6 Thevenin Equivalent for Dispatching

As an example, suppose a new load P_k is connected to bus k. The increment of system production cost C_k for supplying the load is

$$C_k = C_{ko} + \int_o^{P_k} \alpha_k(p)\,dp \tag{24}$$

As an aside it should be noted that the BIC's at all buses other than k also are changed, otherwise there could not be any change in α_k. However the differential power dp at all other buses would be zero so there should be no change in the cost at the other buses.

From Figure 6, application of the potential equation yields

$$\alpha_k = \alpha_{ko} + HP \tag{25}$$

Substituting α_k into Eqn (24), the production cost of the generators in the system for supplying the additional load, P_k, is

$$C_k = C_{ko} + \alpha_{ko}\, P_k + 1/2\ HP_k^2 \tag{26}$$

where C_{ko} is the added no-load cost of the newly committed generators as the result of adding extra P_k load.

In the case of installing a new generator, the saving of production cost owing to the operation of the new generator can be calculated in a similar way. The production cost C_g of the newly installed generator in terms of its power P_g is equal to

$$C_g = a_g P_g^2 + b_g P_g + c_g \tag{27}$$

The reduction of the production cost of the other generators in the system due to the loading shift to this new generator is equal to

$$C_s = C_{so} + \int_o^{P_e} \alpha_k(p)\ dp \tag{28}$$

where C_{so} is the additional saving of no-load costs of some generators taken off-line due to the addition of this new generator. α_k is the BIC of the terminal bus of the new generator. The net saving is equal to

$$\Delta C = C_s - C_g \tag{29}$$

Eqns (24)-(29) provide the basis for quick planning alternative evaluations.

It becomes obvious that, from the ideal viewpoint of system economy, a new load should be connected to a bus where the BIC is as low as possible, and a new power source should be installed at a point where the BIC is as high as possible to get a better economic effect.

From the same reasoning, while considering a potential transaction, the tie line should connect from the sellers system at a low BIC bus to the buyers system at a high BIC bus to achieve the most benefit possible. Similar to Eqns (24)-(29), the increment of the production cost of the sellers system C1, and the decrement of the production costs of the buyers system C2 are respectively equal to

$$C1 = C_{10} + \int_{o}^{P_e} \alpha_1(p)\ dp \tag{30a}$$

$$C2 = C_{20} + \int_{o}^{P_e} \alpha_2(p)\ dp \tag{30b}$$

The net saving to these two systems is

$$\Delta C = C2 - C1$$

$$= C_{20} - C_{10} + \int_{o}^{P_e} [\alpha_2(p) - \alpha_1(p)]\ dp \tag{31}$$

where P_e is the net exchange power at the boundary bus.

These formulae can provide the basis for transaction evaluation.

When representing these systems (buyer and seller) by their Thevenin equivalents (Figure 7), and neglecting nonlinearities of the network model, there results $\alpha_1 = \alpha_{10} + H_1 P$ and $\alpha_2 = \alpha_{20} - H_2 P$.

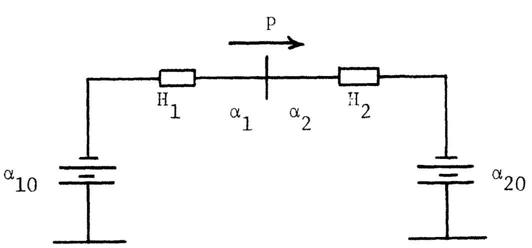


Fig. 7 Equivalent of the transaction systems

The power pool net saving is

$$\Delta C = C_{20} - C_{10} + \int_{o}^{P_e} [\alpha_{20} - \alpha_{10} - (H_2 + H_1)p]dp \tag{32}$$

The maximum saving would be realized when the transaction power is equal to

$$P_e = \frac{\alpha_{20} - \alpha_{10}}{H_2 + H_1} \tag{33}$$

NUMERICAL RESULTS

Results are shown for a system consisting of the major transmission and generation components of an operating power system. The system consists of 24 buses, 14 generators and 32 lines, as shown in Fig. 8.

Case A is a circumstance simply to compare the results of the bus incremental cost dispatch method (BIC) with a Reference method. The Reference method uses B-coefficients to obtain the generation dispatch, which is then input into a full a-c load flow to obtain transmission line flows. Table 1, Case A shows the comparative generation dispatch results. Table 2, Case A shows the resultant active power line flows.

Case B is a circumstance where the transmission line from bus 11 to bus 15, which had a line flow of 136.1 MW in Case A, was given a constraint level of 100 MW. Since the Reference method is not able to treat transmission line constraints, Table 1, Case B shows only the results of the bus incremental cost (BIC) dispatch. Table 2, Case B compares the active power transmission line flows obtained directly from the BIC method with the active power line flows obtained from a full a-c load flow with the BIC generation schedules as input.

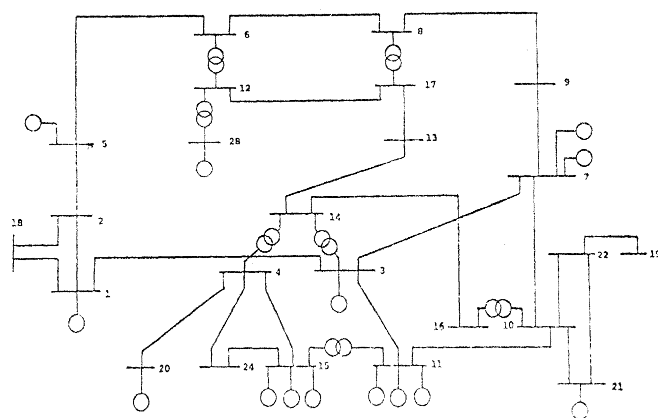


Fig. 8 Single line diagram of sample system

TABLE 1 ECONOMIC GENERATION SCHEDULES

GENERATOR/BUS IDENTIFICATION	CASE A		CASE B
	BIC	REF.	BIC
28	426.0	426.0	426.0
3	147.0	147.0	147.0
5	74.0	74.0	74.0
1	48.0	48.0	48.0
15	148.8	148.8	135.2
15	148.8	148.8	135.2
15	148.8	148.8	135.2
7	9.9	9.9	9.9
7	49.5	49.5	49.5
11	13.0	13.0	13.0
11	64.6	64.9	85.0
11	65.2	66.1	85.6
20	0	0	0
21	0	0	0

TABLE 2 RESULTANT TRANSMISSION LINE FLOWS

LINE IDENTIFICATION		CASE A		CASE B	
FROM BUS	TO BUS	BIC	REF.	BIC	LOAD FLOW
1	2	-3.3	-2.8	-3.2	-2.7
1	3	24.0	23.6	23.9	23.5
1	18	9.0	8.9	9.0	8.9
2	18	11.3	11.4	11.2	11.3
3	7	4.5	4.3	4.3	4.0
4	15	-180.9	-181.2	-178.2	-178.7
4	20	359.2	359.5	359.2	359.5
4	24	-129.0	-129.6	-127.1	-127.4
5	2	14.5	14.2	14.5	14.1
5	6	-51.8	-51.6	-51.7	-51.5
6	8	63.9	64.1	64.0	63.8
9	7	-1.2	-1.2	-1.4	-1.4
9	8	-71.8	-71.8	-71.6	-71.8
10	7	-62.8	-62.4	-62.4	-62.0
11	3	0.6	0.9	3.2	3.5
11	10	46.0	46.0	48.0	47.9
12	17	194.8	194.3	194.8	195.2
13	14	92.1	91.5	92.4	92.0
13	17	-93.8	-93.4	-94.1	-93.9
16	14	-86.0	-86.6	-84.5	-85.1
10	22	11.7	11.7	11.7	11.6
21	22	-8.7	-8.6	-8.7	-8.6
21	10	-21.5	-21.6	-21.5	-21.6
24	15	-129.2	-129.8	-127.3	-127.6
12	28	-326.0	-326.0	-326.0	-326.0
4	14	-51.3	-50.5	-55.9	-55.2
12	6	127.8	127.8	127.7	127.3
14	3	-47.0	-47.3	-49.8	-50.0
15	11	136.1	135.3	100.1	100.2
16	10	84.7	85.2	83.1	83.7
17	8	96.6	96.5	96.3	96.7
22	19	3.0	3.0	3.0	3.0

CONCLUSIONS

1. The network model of economic dispatch exploits the simple but meaningful relationships between bus incremental costs and generation schedules.

2. The network model of economic dispatch clearly shows the relationship between bus incremental costs and line flow direction (from lower to higher incremental costs) as well as line flow magnitude.

3. System configurations with non-uniform X/R ratios in closed loops can be handled by making use of the loop equality constraints that the sum of voltage phase angle differences around a closed loop equals zero.

4. Transmission systems with non-unform X/R ratios, as a consequence, have non zero values of the dual variable of the loop constraint equations. This leads to an effect which can be viewed as the circulating power in the loop required to change the associated line flows from the economically-based determination based on line resistance to the electrically-based determination based on line reactance.

5. The value of the dual variable gives a relative indication of the value of installation of a phase shifter to bring line flows back to the economically-based determination based on line resistance.

6. An area system can be reduced to a Thevenin equivalent for economic dispatching by means of the network model.

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